

ARE YOU READY FOR AP CALCULUS BC?**SCHS EDITION – MR. RIZZI (@CRIZZI @ROCHESTER.K12.MI.US)****Skill #1 – Writing Equations of Lines**1. Write equations for the following lines in point slope form:(a) the line through $(-1, 3)$ and $(2, -4)$

$$y = -\frac{7}{3}(x + 1) + 3 \text{ or } y = -\frac{7}{3}(x - 2) - 4$$

(b) the line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$

$$y = -\frac{3}{2}(x + 1) + 2$$

(c) the line through $(2, 3)$ and parallel to $y = \frac{1}{2}x - 5$

$$y = \frac{1}{2}(x - 2) + 3$$

Skill #2 – Graphing Solutions on a Number Line

2. Graph the solutions to the following inequalities on a number line.

(a) $2x + 3 > 8x - 21$

$$x < 6$$

(b) $x^2 + 7x - 30 \geq 0$

$$x \geq 3 \text{ or } x \leq -10$$

(c) $x^2 - 8x \leq -12$

$$2 \leq x \leq 6$$

Skill #3 – Factoring and Solving Quadratic and Polynomial Equations

3. Factor completely to find all real solutions:

(a) $x^2 + 2x - 3 = 0$

$$x = -3, 1$$

(b) $x^2 + 11x + 24 = 0$

$$x = -8, -3$$

(c) $4x^2 + 12x + 3 = 0$

$$x = -\frac{3}{2} \pm \frac{\sqrt{6}}{2}$$

(d) $x^6 - 16x^4 = 0$

$$x^4(x - 4)(x + 4) = 0$$

$$x = -4, 0, 4$$

(e) $4x^3 - 8x^2 - 25x + 50 = 0$

$$(2x - 5)(2x + 5)(x - 2) = 0$$

$$x = -\frac{5}{2}, \frac{5}{2}, 2$$

(f) $8x^3 + 27 = 0$

$$(2x + 3)(4x^2 - 6x + 9) = 0$$

$$x = -\frac{3}{2}$$

Skill #4 – Evaluating Trigonometric Functions

4. Without using a calculator, evaluate the following in RADIANS.

(a) $\cos\left(\frac{7\pi}{6}\right)$

$$-\frac{\sqrt{3}}{2}$$

(b) $\sin\left(\frac{5\pi}{4}\right)$

$$-\frac{\sqrt{2}}{2}$$

(c) $\tan^{-1}(-1)$

$$-\frac{\pi}{4}$$

(d) $\sin^{-1}(-1)$

$$-\frac{\pi}{2}$$

(e) $\cos\left(\frac{9\pi}{4}\right)$

$$\frac{\sqrt{2}}{2}$$

(f) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\frac{\pi}{3}$$

(g) $\tan\left(\frac{5\pi}{6}\right)$

$$-\frac{\sqrt{3}}{3}$$

(h) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$$\frac{3\pi}{4}$$

Skill #5 – Using a Graphing Calculator to Solve Equations

5. Some equations cannot be solved by hand. Please use the graphing features of your graphing calculator (**in radians**) to solve the following equations. **Give all answers correct to three decimal places.** (Hint: you will need to use 2nd → CALC → Intersect or Zero to do these.)

(a) $5x^3 - 3x^2 - 2x + 1 = 0$ (b) $e^{2-x^2} = 0.5x^2$ (c) $\sin(2x + 1) = 4x^2$ on $[-2, 2]$

$x = -0.606, 0.421, 0.785$

$x = -1.414, 1.414$

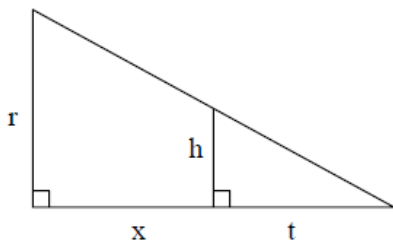
$x = -0.307, 0.481$

6. Find the point of intersection of the lines $3x - y - 7 = 0$ and $x + 5y + 3 = 0$

$(2, -1)$

Skill #6 – Setting Up and Solving Story Problems

7. Express x in terms of the other variables in the picture:



Set up proportion using similar triangles

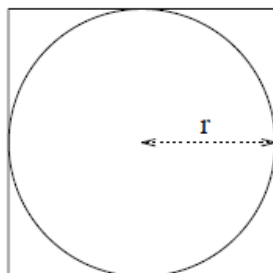
$$x = \frac{t(r-h)}{h}$$

8. Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below.

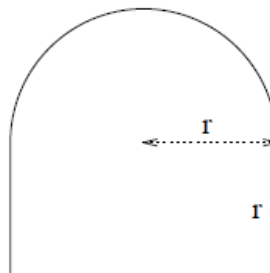
Area of square = $4r^2$

Area of circle = πr^2

ratio = $1 - \frac{\pi}{4}$



(a)



(b)

9. Find a formula for the perimeter of a window of the shape in the picture (b) above.

Perimeter = $4r + \pi r$

10. A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle), what is the surface area of the top of the water?

$$\frac{9\pi}{4}$$

11. Two cars start moving from the same point. One travels south at 100km/hour, the other west at 50 km/hour. How far apart are they two hours later?

$$100\sqrt{5} \text{ km}$$

12. A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the ground?

$$\frac{\pi}{6} \text{ or } 30^\circ$$

For #13-15, use a **graphing calculator** to solve. Give all answers correct to **three decimal places**.

13. The number of students in a school infected with the flu t days after exposure is modeled by the function $P(t) = \frac{300}{1+e^{4-t}}$.

(a) How many students were infected after three days?

(b) When will 100 students be infected?

$$\begin{aligned} \text{a. } & 80.682 \text{ students} \\ \text{b. } & 3.307 \text{ days} \end{aligned}$$

14. Exponential growth is modeled by the function $A = Pe^{kt}$. A culture contains 500 bacteria when $t = 0$. After an hour, the number of bacteria is 1200.

(a) How many bacteria are there after four hours?

(b) After how many hours will there be 8000 bacteria?

$$\begin{aligned} \text{a. } & 16,588.8 \text{ bacteria} \\ \text{b. } & 3.169 \text{ hours} \end{aligned}$$

15. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals, as shown. What dimensions should be used so that the enclosed area will be a **maximum**?

$$\text{Function: } \text{Area} = \frac{200x - 3x^2}{2}$$

$$\text{Dimensions: For each corral length} = \frac{100}{3} \text{ ft and width} = 25 \text{ ft}$$



Skill #7 – Using the Difference Quotient

16. For the following, simplify $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. Show all work.

(a) $f(x) = 2x + 3$

(b) $f(x) = \frac{1}{x+1}$

(c) $f(x) = x^2 + x$

$$\lim_{h \rightarrow 0} \frac{2(x+h) + 3 - (2x+3)}{h} = 2$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = -\frac{1}{(x+1)^2}$$

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 + x + h - (x^2 + x)}{h} = 2x + 1$$

Hint: Find a common denominator for the two fractions in the numerator of the difference quotient.

Skill #8 – Applying the Power, Product, and Quotient Rules

17. Find the derivative of the following functions using either the power, product, or quotient rule:

(a) $y = 4x^3 + \frac{1}{4}x^2 - 5x + 1$

$$y' = 12x^2 + \frac{1}{2}x - 5$$

(b) $f(x) = (2x^2 + 1)(3x + 4)$

$$f'(x) = (4x)(3x + 4) + (2x^2 + 1)(3) \\ \text{or } f'(x) = 18x^2 + 16x + 3$$

(c) $f(x) = x^{3/2} + 4x^{-1/4}$

$$f'(x) = \frac{3}{2}x^{1/2} - x^{-5/4}$$

(d) $f(x) = \frac{3x+4}{2x^2+5x+1}$

$$f'(x) = \frac{3(2x^2 + 5x + 1) - (3x + 4)(4x + 5)}{(2x^2 + 5x + 1)^2}$$

18. What is the slope of the line tangent to $y = 3x^2 - 2x + 1$ when $x = 2$?

The derivative is $\frac{dy}{dx} = 6x - 2$, so the derivative evaluated when $x = 2$ is 10. Therefore, the slope is 10.

19. Find the x value(s) where the curve has a horizontal tangent line: $f(x) = \frac{1}{3}x^3 - x^2 - 15x + 4$

The derivative is $f'(x) = x^2 - 2x - 15$. Graphs have horizontal tangents when the derivative equals zero, so solving the equation $0 = x^2 - 2x - 15$ will give us the right answers. Therefore, the answers are $x = -3, 5$.

Skill #9 – Limits Review

Please view the series of videos in the section entitled "Limits" in [Differential Calculus](#) on Khan Academy. To gain access to this list of videos, sign up as part of my class on the Khan Academy website. You will receive a grade on the first day of class for watching **ALL of the Limit videos and completing the practice questions and quizzes up through Quiz #3**.

In order to sign up for my class on Khan Academy, please read the following directions:

1. Create a free account at [khanacademy.org](https://www.khanacademy.org) (or log in with your existing accounts)
2. Visit [khanacademy.org/coaches](https://www.khanacademy.org/coaches)
3. In the "Add a coach" field, enter the class code **DCWB8EAB**
4. Complete the videos and practice problems at <https://www.khanacademy.org/math/ap-calculus-bc/bc-limits-continuity>